Team Control Number

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Problem Chosen

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2014 Mathematical Contest in Modeling (MCM) Summary Sheet

Our target is to analyze the driving principle Keep-Right-Except-To-Pass in two aspects: one is the effects of traffic flow; the other is traffic security. As a result, we formulate two methods to analyze a traffic process, which include four models and six input data and output two indexes. Based on that, the results of analyzing the traffic rules from three aspects are given to the driving rules and a suggestion to improve the road driving rules is proposed. We also talk about some future expectations.

Six Parameters and Two Indexes

We analyse six parameters concerned with present traffic flow, which include the length of this road section researched, the probability that various vehicles simultaneously appear in the road section, the speed limit section, the total time of the research process, the frequency that vehicles reach the road section and the friction coefficient of the road. We designed two indexes to analyse this driving rules, which are traffic flow index and the number of vehicles having potential safety hazard.

Two Methods and a Process

In order to analyze the two indexes output in a traffic process, we formulate two technology roads containing the theory analysis and simulation program:

1. Under the condition that six input data are ensured, to acquire the theory value of traffic flow by complex calculation based on the reasonable model hypothesis.
2. In order to verify the veracity of the theory result, we use simulation program to simulate 1000 traffic process in every special situation. After analyzing output traffic flow index in 2000 experiments, we find that the theory results correspond to the results of experiments. Not only are the theory results verified right, but also the validity of the experiment method is ensured. Meanwhile, the experiments count the number of vehicles having potential safety hazard in a simulation process, which provide reliable data to analyze the road security

with different input data.

Four Models and Three Questions

In order to assist these two methods that include the theory analysis and simulation program, we formulate four models in total, which contain theory calculation, lane-changing model, overtaking model and security analysis.

1. Theory calculation deduce a core index, that is, the function relation between the theory value of traffic flow and six input data.
2. With the purpose of achieving simulation program method, we design the lane-changing model and overtaking model. We analyze and discuss the process of achieving Changing Method and overtaking Method packed by the Car Class in the simulation process in detail.
3. But the security analyze model is designed to obtain the number of vehicles with potential safety hazard in each simulation. Also, we talk about the detailed operation that how such a model is successfully applied to program simulation.

First of all, we analyze the question that both traffic flow and security are low when traffic rule Keep-Right-Except-To-Pass is applied to a heavy traffic road. Then, we analyze the reason why the traffic rule Keep-Right-Except-To-Pass can't be applied to those countries where it is required to drive on the left by simply exchanging left and right. Eventually, we conclude that if we rule out certain inevitable factors, we can acquire high traffic flow and few vehicles with potential safety hazard when the traffic rule Keep-Right-Except-To-Pass is applied to an intelligent control system.

One Suggestions and Some Prospects

On the basis of three questions above, we analyze and put forward a suggestion to improve the traffic rule Keep-Right-Except-To-Pass according to two indexes including the velocity gradient and vehicle type. Ultimately, we also roughly discussed some prospects of control system about using of unmanned technology in the future.

A Study in Keep-Right-Except-To-Pass Rule

Abstract:

The purpose of this paper is to analyze the road traffic flow and security problem under the driving rule Keep-Right-Except-To-Pass by formulating math model. Moreover, based on reasonable hypothesis, we use simulation data to represent the character of act rules after considering the advantages and disadvantages. Finally, we set forth and analyze different rule choice.

At the beginning, we analyze the road with many vehicles and road with few vehicles respectively. Taking six indexes into account which includes the length of this road section researched, the probability that various vehicles appear in the road section, the speed limit section, the total time of the research process, the frequency that vehicles reach the road section and the friction coefficient of the road, we deduce a theory index that can measure the traffic flow performance. Meanwhile, we use simulation program to simulate 1000 traffic process in every special situation. After analyzing output traffic flow index in 2000 experiments, we find that the theory results correspond to the results of experiments. It is also concluded that this traffic rule may lead to low traffic flow and low security in the heavy traffic road, for which we set forth a suggestion to improve according to the velocity gratitude and vehicle type.

Then, after analyzing the present research situation at home and abroad, we continue to use the data acquired by the math model combined with program simulation and we conclude that the traffic rule Keep-Right-Except-To-Pass can't be applied to those countries where it is required to drive on the left by simply exchanging left and right. Furthermore, the traffic flow of countries where it is required to drive on the left is bigger than that of countries where it is required to drive on the right, but the number of vehicles with potential safety hazard of the former is larger.

Finally, combined with former models, we discuss that if we rule out certain inevitable factors, we can acquire high traffic flow and few vehicles with potential safety hazard when the traffic rule Keep-Right-Except-To-Pass is applied to an intelligent control system. In addition, we broadly talk about the future exception about applying the unmanned control system in the future.

Keywords: Mean Traffic Flow, Lane changing model, Overtaking model Safety analysis, The Monte Carlo method

Contents

I. Introduction

1.1 The Couse

Artificial intelligence has become one of the greatest advances of the 21st century. It can make the conventional traffic model safer, more efficient and energy save if we design an intelligent-control system and make up some perfect road transport rules. There is no doubt that this system will be a main stream in the future. In the system, the vehicle can use its own control system to realize the unmanned or assistant-drive and the traffic facilities can monitor and adjust the massage of the road condition by the intelligent monitor system.

The traffic rules formulated by different coutries in the world are diverse. In many countries such as the U.S.A., China and so on, it is required that vehiclees run on the right and overtake the other from left, while the rules in some countries like are opposite.

Thus, developing a relatively perfect traffic rules are not only the first step in intelligent control systems and even unmanned technology, but also the most important step. A good set of criterion consider safe driving the first meaning and maximize the traffic flow.

1.2 Issues Raised

This paper aims through theoretical analysis and simulation programming comparative analysis of two approaches in order to address the following issues:

- When analyzing fewer vehicles and more sections, consider the speed, security, and other factors on the highway, and establish a mathematical model which is able to measure traffic performance, to evaluate whether this rule could increase the traffic flow by "Keeping-Right-Except-To-Pass"; if not , then modify the rules.
- Investigate whether the solution of the first problem can be applied directly on the countries where cars keep to the left by simply changing the direction.
- Analyze that under the control of the intelligent system, how the traffic flow will be affected in the solution of the first problem.

II. Preparation

2.1 Common Sense

In most countries where the vehicles run on the right, the highway is usually the two-way four-lane road and a one-way two-lane road, that is, a super-lane carriageway.

The leftmost lane is super, the middle lane is a way, and emergency lane at the far right is an emergency use. According to the traffic rules, vehicles will be traveling in the driveway, and long-term occupation of ultra-driveway is not permitted. Therefore, under normal circumstances, overtaking the other car is from the left surpass lane.

Figure 2.1.1 Highway Traffic Rules

2.2 Global Assumption

- Without considering the occurred accident is because of its own breakdown.
- Without considering the occurred accident is because of human's wrong judgment which was influenced by the weather condition.
- Without considering too big or small vehicles whose speed are beyond the limited range.
- Without considering the driver malicious behavior.
- Without considering the freeway road‘s individual characteristics impact on traffic safety.
- Without considering the drivers who are nervous, stunned, or just surrender

2.3 Symbol and Parameters

2.3.1 Symbol

Symbol	Explain
t	Observation period $[0,t)$ time value
$N(t)$	In time interval $[0,t)$, the number of vehicles which entered lane

Q	Road flow
$\bar{v}$	Mean Velocity Interval
$t^{(i)}$	The total time that vehicle i spent in running in the road whose length is L
$\bar{T}$, $E\left(t^{(i)}\right)$	The average time by all vehicles spent on through the road
L	Lane length
$P_n\left(t_1, t_2\right)$	The probability of n vehicles to arrive in the time interval $\left[t_1, t_2\right)$
K	Traffic Flow Density
h_d	Space Headway
h_t	Time Headway
$C_i, i=1,2,\cdots,6$	Vehicle models
$L_{C_i}, i=1,2,\cdots,6$	Vehicle model i
$P\left(C_i\right), i=1,2,\cdots,6$	The probability that vehicle model i in the road
$v^{(i)}$	An instantaneous speed of vehicle model i
s_{safe}	The range of safety Vehicle distance
$V^{(light)}$	Under the Light traffic, the speed limit range
$V^{(heavy)}$	Under the Heavy traffic, the speed limit range
s	Safety distance between adjacent vehicles
μ	The friction coefficient of the vehicle and the road surface
$E\left(Q\right)$	The average vehicle flow

2.3.2 Traffic Flow Parameters

The document [1] has expounded and defined a series of judging the element parameter of the traffic flow, and discussed the road character of the highway and its abi-lity to pass the road in details. We will use some main parameters as the judgment of the road traffic ability in this modeling.

Then, we begin to state its basic meaning.

1. Flow

Flow Q points that, in a given time unit, the total number of vehicles through a site, a section or a lane is named flow or traffic volume

$$Q = \frac{N(t)}{t} \tag{2.3.1}$$

2. Mean Velocity Interval

Velocity interval, which is called the driving speed, is what the traffic miles div-ide the time needed to travel through the section (including the parking time).

Velocity interval is a composite indicator, used to evaluate the unobstructed degree of a road, and estimate the situation of traffic delays. When we studying traffic capacity, mean velocity interval is used as the speed standard. Mean Velocity Interval is the ratio between the journey of this road section and the average time $\bar{T}$ spent by all vehicles:

$$\bar{v} = \frac{L}{\bar{T}} = \frac{L}{\frac{1}{N(t)} \sum_i t^{(i)}} \tag{2.3.2}$$

Among it, the vehicle i had spent $t^{(i)}$ in the road section whose length is L.

3. Mean Traffic Flow Density

Vehicle flow density refers to number of a certain instantaneous vehicle a lane or a direction in the length of the road. It reflects the proximity between traffic stream vehicle degree, Sometimes it also can be represented by the total length of all vehicles on the known road and the road length ratio. Lane space percentage show that:

$$K = \frac{N(t)}{L}$$

In the simulation experiments, K expectation is referred to as **the average density**.

4. Space Headway

Space headway h_d is in a motorcade traveling in the same direction, the two adjacent vehicles head space distance or interval.

5. Time Headway

Time headway h_t is referring to use the time to express Space headway h_d, also called Time headway.

We can easily reach the conclusion that the relation of Time Headway, Space Headway and the velocity is:

$$h_d = \frac{v^{(i)}}{3.6} h_t$$

2.3.3 Traffic flow Characteristics

We study the properties of traffic flow for a road whose length is L. It mainly includes three aspects: The process of vehicles entering the road; Running process of the vehicle on the road; the process that vehicle pass this road.

In 3.1.2, we will write code to input data on the basis of Traffic flow characteristics.

1. Input Process

Use $N(t)$ to Show the number of vehicle road into the time interval $[0,t)$, and let $P_n(t_1,t_2)$ represent the probability of n vehicles to arrive in the time interval $[t_1,t_2)$.

A. Light traffic

Under the light traffic, also the small vehicle flow density, and there is almost no outside interference factors, so we make objective assumptions on the vehicle that appeared in the road:

(1) In the number of non-overlapping time intervals vehicles entering the road sections are independent of each other,

(2) For sufficiently small time Δt, the probability about a vehicle run and must run in lane carriageway is irrelevant for t in the time interval $[t_1, t_2)$, that is

$$P_1(t, t+\Delta t) = \lambda \Delta t + o(\Delta t)$$

Among it, for $o(\Delta t)$, When $\Delta t \to 0$, $o(\Delta t)$ is A higher order infinitesimal about Δt , λ is the number of vehicles entered into the road in a unit of time.

(3) For sufficiently small time Δt, the probability about one or two more vehicles into the lane section is too small to be Neglected in the time interval $[t_1, t_2)$, that is

$$\sum_{n=2}^{\infty} P_n(t, t+\Delta t) = o(\Delta t)$$

According to the above three hypotheses, we are not difficult to obtain

$$P_n(0,t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, n = 1, 2, \cdots$$

It shows that under the light traffic, the Input process obeys the Poisson distribution in the fixed section. Among it, λ as the average arrival rate parameters.

B. Heavy Traffic

Under the heavy traffic, the traffic is crowded, free driving the opportunity not to be many. Then vehicles that entered the road are independently of each other, it can be regarded as a independent repeated trials in essence. At this time, there is

$$P_n(0,t) = \binom{k}{n} \left(\frac{\lambda t}{k}\right)^n \left(1 - \frac{\lambda t}{k}\right)^{k-n}, n = 1, 2, \cdots$$

In other words, under the heavy traffic, the Input process obeys the binomial distribution in the fixed section. Among it, λ as the average arrival rate parameters.

C. Vehicle models

During input process, We enumerate six kinds of vehicle models as input[9], as shown in **Table 2.3.1**, as a matter of convenience, we labeled different models as $C_i, i = 1, 2, \cdots, 6$ in turn. Of which the total car lengths are respectively labeled $L_{C_i}, i = 1, 2, \cdots, 6$, its appearance probability is denoted by $P(C_i), i = 1, 2, \cdots, 6$.

Table 2.3.1 Outline dimensions of vehicle models

Project / Vehicle models	Outline dimensions(m)		
	Total length	Total width	The total height
Mini car C_1	3.50	1.60	1.80
Car C_2	4.80	1.80	2.00
Light car C_3	7.00	2.10	2.60
Mid-size car C_4	9.00	2.50	3.20
Large bus C_5	12.00	2.50	3.20
Large truck C_6	10.00	2.50	4.00

2. Running Process

A. Moving Status

In the process of driving in road whose length is L, we assume that the driver in the driving only three kinds of demand: **change**, **overtaking**, **maintain**.

(1) In the traffic lane, the driver does not exist overtaking behavior, at this time to drive the vehicle speed are the stable value, If traffic is relatively poor, have more chance to free exercise, we consider the lane changing model in 3.2.

(2) But when traffic flow is crowding, and chances for free driving is less, we pretend that drivers will neither overtake nor change lanes, just keeping lanes.

(3) When drivers produce the overtaking demand, we consider the overtaking model in 3.3.

B. Speed Limited

The velocity limit during the process of driving can be classified into two cases. Under the light traffic, we set the speed limit range is $V^{(light)}$, or $\left[v_{\min}^{(light)}, v_{\max}^{(light)}\right]$. And under the heavy traffic, we set the speed limit range is $V^{(heavy)}$, or $\left[v_{\min}^{(heavy)}, v_{\max}^{(heavy)}\right]$. To ensure that the result is true and reliable, we can suppose the velocity is $v^{(i)} \in \left[V_{\min}^{(light,heavy)}, V_{\max}^{(light,heavy)}\right]$ and the section satisfy "2σ", that is:

$$v^{(i)} \sim N\left(\frac{1}{2}\left(V_{\min}^{(light,heavy)} + V_{\max}^{(light,heavy)}\right), \frac{1}{4}\left(V_{\max}^{(light,heavy)} + V_{\min}^{(light,heavy)}\right)\right)$$

C. Safety distance

The Safety distance between adjacent vehicles depends on both the drivers' reaction time and the cars' braking performance.Breaking mainly depends on the frictional force between the tire and the ground the size of the friction depends on the coefficient of friction. Pretending that the coefficient of friction is μ , the friction distance is:

$$s_{brake} = \frac{\left(v^{(i)}\right)^2}{2g\mu}$$

The coefficient of friction is related to many factors. The general value is 0.8 or so, failing under 0.2 when raining, and even less when driving on the snow and ice road.

The reaction time of ordinary people is up to 0.2 seconds. when considering the cars' response time, we pretend that it takes 0.2 to 2 seconds to finish the total action .So the safety distance between adjacent vehicles s is：

$$s \in s_{safe} = \left[\frac{\left(v^{(i)}\right)^2}{2g\mu} + 0.2v^{(i)}, \frac{\left(v^{(i)}\right)^2}{2g\mu} + 2v^{(i)} \right]$$

It may be that the safety distance meet "3σ rule" in this interval, so the safety distance between adjacent vehicles satisfies the relationship:

$$s \sim N\left(\frac{\left(v^{(i)}\right)^2}{2g\mu} + 1.1v^{(i)}, \frac{11v^{(i)}}{30} \right);$$

Safety space headway satisfies the relationship:

$$h_d - L_{C_i} \sim N\left(\frac{\left(v^{(i)}\right)^2}{2g\mu} + 1.1v^{(i)}, \frac{11v^{(i)}}{30} \right)$$

Safety time headway satisfies the relationship:

$$\frac{v^{(i)}}{3.6} h_t - L_{C_i} \sim N\left(\frac{\left(v^{(i)}\right)^2}{2g\mu} + 1.1v^{(i)}, \frac{11v^{(i)}}{30} \right)$$

Calculate part of the vehicle braking distance , as shown in **Table 2.3.2**, we can see that, The braking distance is generally controlled at 30~150 on the highway.

Table 2.3.2 Part of the vehicle braking distance

Speed(km/h)	60	70	80	90	100
Braking distance(m)	34.4	43.5	53.7	64.9	77.0
Speed(km/h)	**120**	**150**	**180**	**200**	**250**
Braking distance(m)	104.2	152.4	209.4	252.4	377.0

3. Output Process

- Safety index: We utilize the relation between the rate of accident and the traffic flow as the index of security assessment.
- Traffic flow index: the traffic flow is defined in the **Formula 2.3.1**. In some special road sections, we only use this index to measure the traffic flow performance. Generally, we use the expectation of traffic flow $E(Q)$ as an index to measure the a road section.

III. Models

3.1 Framework Design

A good packaging design can help us comprehend the function of the program quickly and directly, and we expect the design have the black box which has the features shown in Paint 3.1.1. We just need to input the specific parameters, and we will get the output we need:

$$\left(E(Q), Safety\_Index\right) = TrafficProcess\left(L, P(C_i), V^{(light,heavy)}, t, \lambda, \mu\right)$$

Figure 3.1.1 framework map of test the black box

We can feel in the above design, $E(Q)$ is $L, P(C_i), V^{(light,heavy)}, t, \lambda, \mu$'s function.

Theoretically speaking, it is impossible to cause accident under the situation that the driver absolutely obeys the traffic rules. Consequently, the security index has to be acquired via the simulation program.

3.1.1 Theoretical Analysis

The following derivation is about $E(Q)$.

First of all, it's easy to calculate the average length is

$$E(L_C)=\sum_{i=1}^{6}P(C_i)L_{C_i}$$

We can know by the **Formula 2.3.2**,

$$E(v)=\frac{L+s}{\frac{1}{N(t)}\sum_i t^{(i)}}=\frac{1}{6}\sum_{i=1}^{6}\frac{L+\frac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}}{\frac{1}{N(t)}\sum_i t^{(i)}}=\frac{1}{6}\sum_{i=1}^{6}\frac{L+\frac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}}{\frac{2L}{V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}}}$$

So, the number of vehicles about the average time that all vehicles passing through the road is

$$N_{out}=\frac{E(L_C)}{E(v)}=\frac{6\sum_{i=1}^{6}P(C_i)L_{C_i}}{\sum_{i=1}^{6}\frac{L+\frac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}}{\frac{1}{N(t)}\sum_i t^{(i)}}}=\frac{6\sum_{i=1}^{6}P(C_i)L_{C_i}}{\sum_{i=1}^{6}\frac{L+\frac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}}{\frac{2L}{V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}}}}$$

Obviously, according to the hypothesis of the input process above, when driving out, for a time Δt that is short enough, in the time range $[t,t+\Delta t)$, the probability that a vehicle leaves the road section does not relate to the t and is proportional to the length of the range Δt, that is:

$$P(time\le\Delta t)=N_{out}\Delta t+o(\Delta t)$$

We realize that at the moment $t+\Delta t$, the condition that there are n vehicles in the road section is converted from the condition below:

(1) There are n vehicles at the moment t and no vehicles enter or leave, whose proportion is:

$$1-\lambda\Delta t-N_{out}\Delta t+o(\Delta t)$$

(2) There are $n+1$ vehicles at the moment t and no vehicles enter but a vehicle leave, whose proportion is:

$$N_{out}\Delta t+o(\Delta t)$$

(3) There are $n-1$ vehicles at the moment t and no vehicles leave but a vehicle

enter, whose proportion is：

$$\lambda\Delta t + o(\Delta t)$$

According to the hypothesis, there are only three conditions above, and the proportion of other condition is too small to be considered. At the moment $t+\Delta t$, the proportion that there are n vehicles in the road section is

$$P_n(0,t+\Delta t) = P_n(0,t)(1-\lambda\Delta t - N_{out}\Delta t) + P_{n+1}(0,t)N_{out}\Delta t + P_{n-1}(0,t)\lambda\Delta t + o(\Delta t)$$

When $n=0$, otherwise

$$P_n(0,t+\Delta t) = P_n(0,t)(1-\lambda\Delta t) + P_1(0,t)(1-\lambda\Delta t)N_{out}\Delta t + o(\Delta t)$$

Make $\Delta t \to 0$, and we will notice that $\frac{dP_n(0,t)}{dt} = 0$(Because the probability is not connected with time), there are the following equations

$$\begin{cases} \lambda P_{n-1}(0,t) + N_{out}P_{n+1}(0,t) - (\lambda + N_{out})P_n(0,t) = 0 \\ -\lambda P_0(0,t) + N_{out}P_1(0,t) = 0 \end{cases}$$

We get

$$P_n(0,t) = \left(\frac{\lambda}{N_{out}}\right)^n P_0(0,t)$$

Notice

$$\sum_{n=0}^{\infty} P_n(0,t) = 1$$

If $\frac{\lambda}{N_{out}} < 1$, there are

$$P_0(0,t) = 1 - \frac{\lambda}{N_{out}},\ P_n(0,t) = \left(1-\frac{\lambda}{N_{out}}\right)\left(\frac{\lambda}{N_{out}}\right)^n$$

If $\frac{\lambda}{N_{out}} \ge 1$, the traffic jam will happen, leave out.

So far, we can easily obtain the number ($N(t)$) of the vehicles that enter the road section during the time section $[0,t)$, which is

$$N(t)=\sum_{n=0}^{\infty} nP_0(0,t)=\frac{\lambda}{N_{out}-\lambda} \tag{3.1.1}$$

Bring N_{out} into the **Formula 3.1.1**, we can get the equation

$$\frac{\lambda}{\dfrac{6\sum_{i=1}^{6} P(C_i)L_{C_i}}{\sum_{i=1}^{6}\dfrac{L+\dfrac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}}{\dfrac{2L}{V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}}}}-\lambda}=N(t)$$

Get:

$$N(t)=\frac{6\sum_{i=1}^{6} P(C_i)L_{C_i}}{\lambda\sum_{i=1}^{6}\dfrac{L+\dfrac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}}{\dfrac{2L}{V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}}}}-1=\frac{2LE(L_C)}{\lambda\left(L+E(s)\right)\left(V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}\right)}-1$$

Finally, according to the **Formula 2.3.1**, we get:

$$E(Q)=\frac{E\left(N(t)\right)}{t}=\frac{2LE(L_C)}{\lambda t\left(L+E(s)\right)\left(V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}\right)}-\frac{1}{t}$$

$$\approx\frac{2LE(L_C)}{\lambda t\left(L+E(s)\right)\left(V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}\right)}$$

3.1.2 Monte Carlo Method

In 2.3.3, we have already roughly introduced the driving behavior on the road. In order to insure the reliability of theoretical result, we try using the Monte Carlo method to have an auxiliary contrast for the theoretical result, to make the result more reliable. Meanwhile, we count the security index from the experiment.

We set fixed parameters for an experiment:

test time t , road length L , car types i , numbers of occurrences $P(C_i)$, the speed limit range $V^{(light,heavy)}$, arrival rate λ, coefficient of friction μ

We set fixed parameters for an experiment: test time t, road length L.

Using the object-oriented design method, we abstract the elements and the process in the total experiment to two objects: cars and roads.

In terms of cars, we use the unified modeling language (UML) to express it. As shown in **Figure 3.1.2**. (As **Figure 3.1.2** shows)

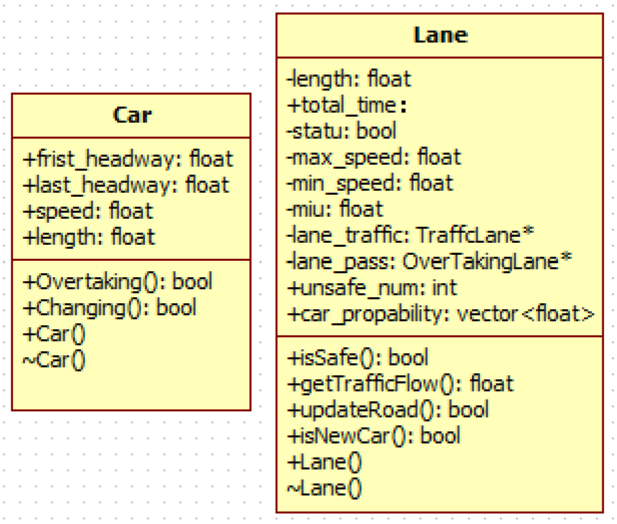

Figure 3.1.2 the Class Diagram of Car Class and Class Diagram of Lane Class

In the process of the simulation, just need to invoke the overtaking method so as to overtake. The realization of this method is shown as 3.3. If overtaking fails, send a message to the road object, letting the object decide whether the experiment will continue or not.

To roads, when analyzing problems, we have pointed out the design features of the highway in most countries people drive on the right, that in essence, there are only two lanes under the condition that we don't consider the emergency lane. In the same way, we use UML to describe the road object as **Figure 3.1.2** shows.

The road object contains a status attribute, also called road safety attribute. In the process of the experiment, check the status in time. If the road status reveals that an accident happens in an experiment, we identify that the experiment fail, and have another one experiment.

Above all, we obtain the design of the whole simulation progress, shown in the **Figure 3.1.3**.

When the road condition is updated, so are the road situation (if the road situation is good), the number of vehicles in the road (the total number of the vehicles at present), the Time Headway and Space Headway of every vehicle (the journey at this time is the journey after changing the direction, which correspond to the probability distribution described in 3.3.2), etc.

Ultimately two values are output: one is the mean discharge and the other is the existence of security problems of vehicle.

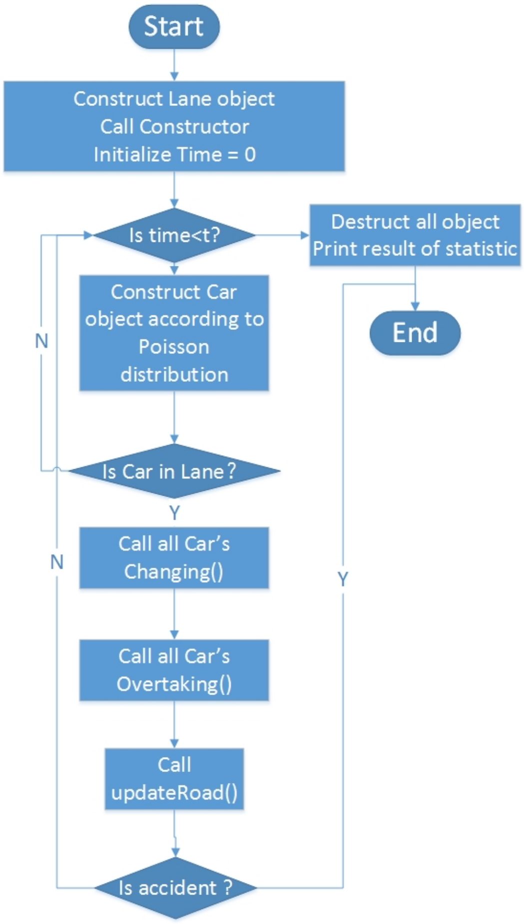

Figure 3.1.3 the Process of main()'s Method Design

3.2 Lane-changing Model

The lane changing module is designed to help the result of simulation program be more true and reliable. In the lane changing module, only the lane changing in carriage-way is considered. You can refer to the surpassing module in 3.3 to realize the lane changing between overtaking lane and carriageway.

To ensure the security, we always consider the situation that vehicle move to the carriageway where the Time Headway is bigger. Due to that, the flow diagram design is shown by the **Figure 3.2.1**.

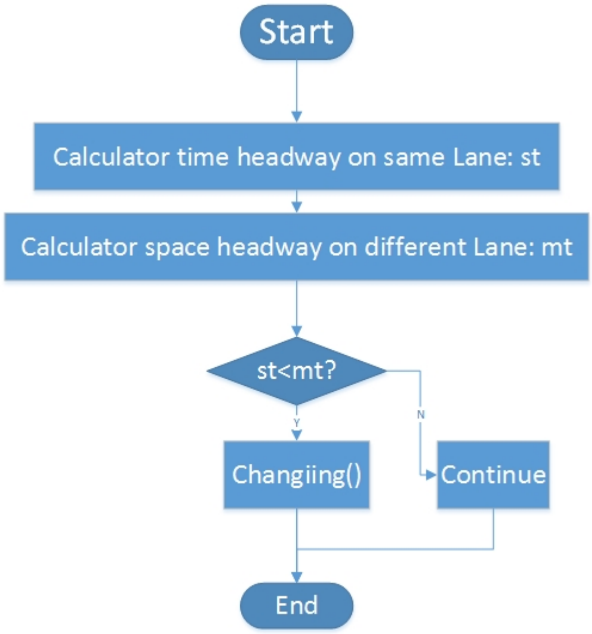

Figure 3.2.1 the Process of Lane Changing Method Design

3.3 Overtaking Model

3.3.1 Overtaking Demand and Overtaking Capability

Drivers want to keep the speed we wants when driving on the road, but the complexity of the traffic composition, the difference among vehicle types and drivers' personalities, result in the big difference in desired speed between the drivers and the cars[2-4], in which some cars are faster and other drives more slowly. When the faster car runs behind the slower car, the fast hopes to keep its own expected speed, so overtaking is in demand. When finding that there is a certain amount of clearance between traffic flow in the overtaking lane and cars in the traffic lane, the vehicle which wishes to overtake starts to run into the overtaking lane to overtake; when the overtaking behavior is finished, the car returns to the driving lane.

As we can see, taking an overtaking behavior on a two-lane highway mainly have two reasons: One is that there is a speed difference between cars, and the overtaking in demand; the other is that traffic flow in the overtaking lane and the place between cars in the traffic lane could offer the capacity needed to overtake and return.

3.3.2 Overtaking Process

The process of overtaking is shown in 3.3.1, Car $n+1$ and Car $n+2$ respectively represent the car ahead and the first car. Car n will have an overtaking behavior; car m is running in the overtaking lane.

We suppose the Time Headway between the front vehicle $n+1$ and overtaking vehicle n is $h_{t(n,n+1)}$; after returning the carriageway, the Time Headway between front vehicle n and overtaking vehicle $n+1$ is $h^*_{t(n,n+1)}$, which should be safe and satisfy the relation:

$$\frac{v^{(i)}}{3.6}h^*_{t(n,n+1)} - L_{C_i} \sim N\left(\frac{\left(v^{(i)}\right)^2}{2g\mu} + 1.1v^{(i)}, \frac{11v^{(i)}}{30}\right)$$

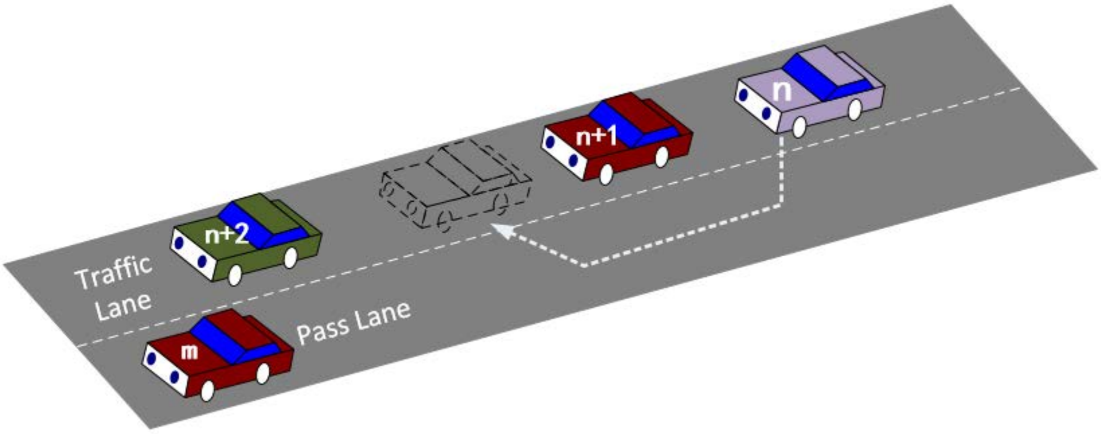

Figure 3.3.1 the Overtaking Process

When Car n overtakes, based on in the overtaking lane Car m is overtaking but not finished yet, and that the cars behind is likely to overtake, considering safety, Car n cannot stay on the overtaking lane for a long period of time, and shall run into the place between Car $n+1$ and Car $n+2$, That is offering enough time headway.

To sum up, the design process of overtaking method as shown in **Figure 3.3.2**.

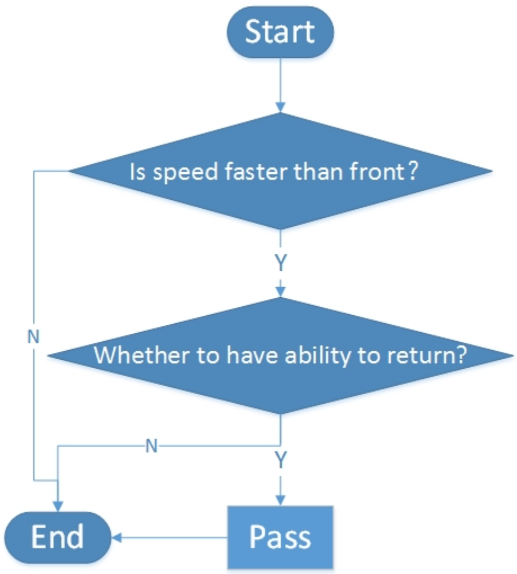

Figure 3.3.2 the Process of Overtaking Method Design

3.4 Security Analysis

Safety analysis is designed to help program to analogize outputting the quantity of the potential safety hazard .Below we construct analysis method for the safety of vehicle operation, analysis of vehicle distance control.

A vehicle is called hidden safety problems, if it meets the following conditions:

$$\frac{v^{(i)}}{3.6}h^{*}_{t(n,n+1)}-L_{C_i}\in\left(0,\frac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}-\frac{11v^{(i)}}{10}\right)\cup\left(\frac{\left(v^{(i)}\right)^2}{2g\mu}+1.1v^{(i)}+\frac{11v^{(i)}}{30},+\infty\right)$$

IV. Solutions

To composite the various module design in Part 3 Modules, we use C++ to program an application. This application, which simulates a road section whose length is L, output every character of traffic flow in a road section during a time cycle which is set as t. The code is referred to the **Appendix 7.1**.

Next we will analyze those three questions in 1.2 in two ways: the theory result and simulation result.

4.1 Keep-Right-Except-To-Pass Performance Analysis

4.1.1 in Light Traffic

1. Theoretical Results

We use the consequence of 3.1 to calculate the process below:

$$E(Q)=TrafficProcess\begin{pmatrix}L=30km,\\ P(C_i)=\{0.15,0.2,0.3,0.2,0.1,0.05\},\\ V^{(light)}=[90,120]km/h,\\ t=240\min,\\ \lambda=k,\\ \mu=0.8\end{pmatrix}$$

Among it $\lambda=k$, $k=1,2,\cdots 10$.

In order to a straightforward, put the result of $E(Q)$ as **Figure 4.1.1**, accurate data refer to **Appendix 7.2**. The changing situation we can see is:

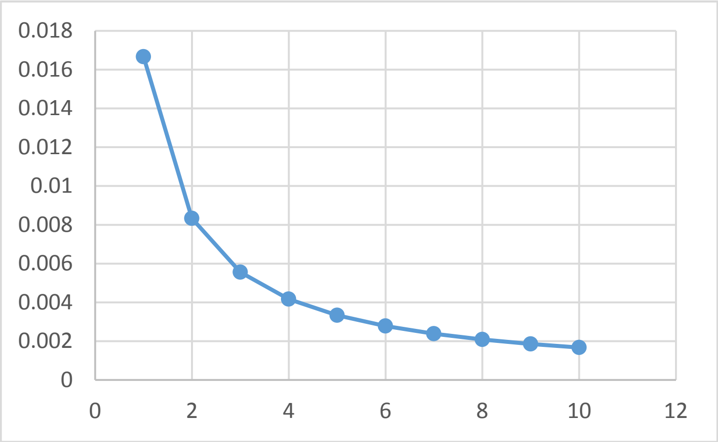

Figure 4.1.1 the Changing Situation of $E(Q)$ Along with the Arrival Rate Changes

2. Simulation Results

Aiming at the inputting data, we simulate 1000 experiments in all. Taking $k=3$ and $k=5$ for example, put the results the program output as the **Figure 4.1.2** and **Figure 4.1.3**.

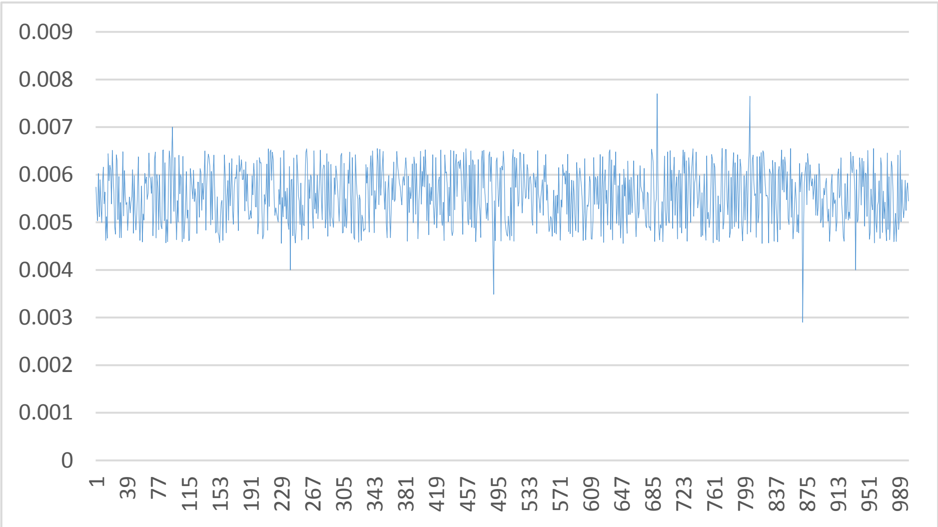

Figure 4.1.2 the Simulation of Results Fluctuation when $k=3$

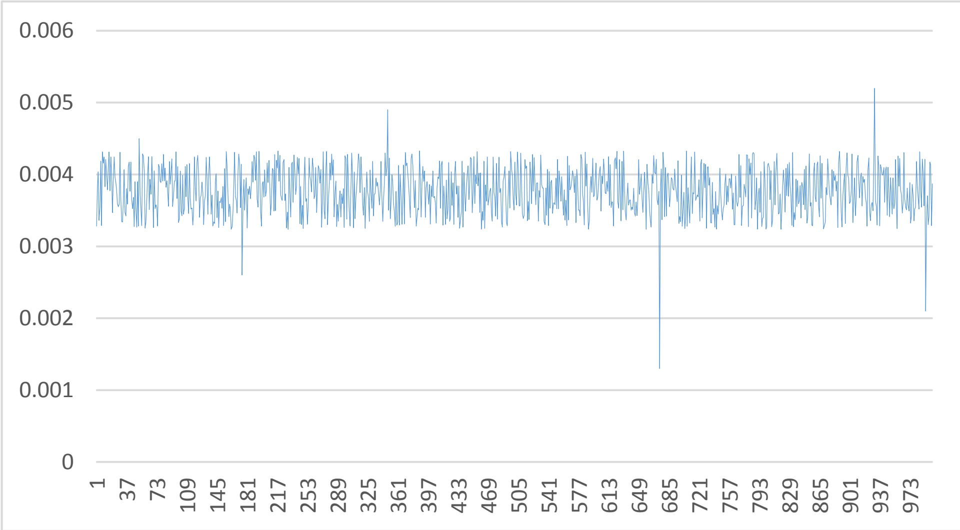

Figure 4.1.3 the Simulation of Results Fluctuation when $k = 5$

It is clear that simulation results are stable over a range, so we believe that the simulation results are coincide with the theoretical results.

The application also output the number of accidents in 1000 experiments. In order to reflect the simulation results directly, a graph is used to show the relation between the number of accidents and the vehicles passing the road section, as shown in the **Figure 4.1.4**.

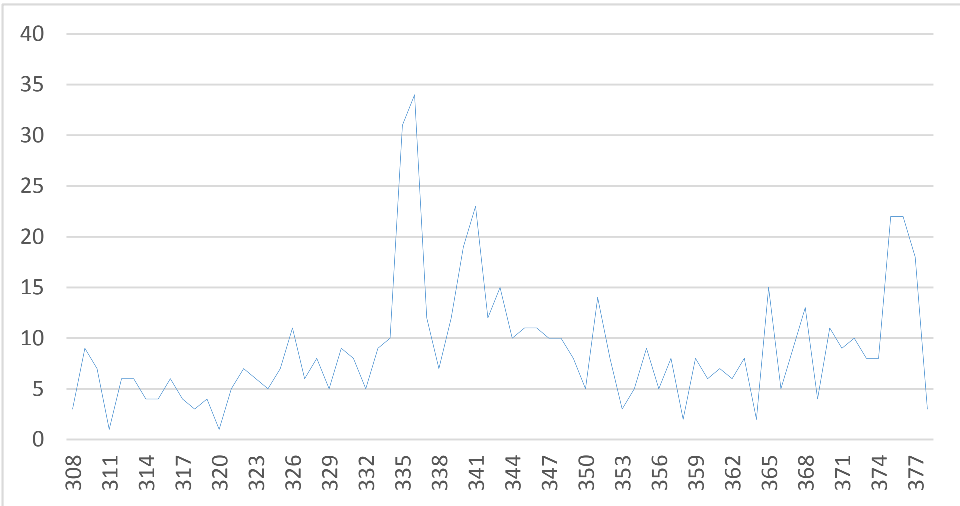

Figure 4.1.4 Relationship between Vehicles existed Security Problems and Traffic

As we can see in **Figure 4.1.4**, the number of hidden safety problems of the vehicle are maintained at below 10 and the fluctuation is stable, so the safety is all right under the light traffic.

4.1.2 in Heavy Traffic

1. Theoretical Results

We use the consequence of 3.1 to calculate the process below:

$$E(Q) = TrafficProcess\begin{pmatrix} L = 30km, \\ P(C_i) = \{0.15, 0.2, 0.3, 0.2, 0.1, 0.05\}, \\ V^{(heavy)} = [50,100]\, km/h, \\ t = 240\min, \\ \lambda = k, \\ \mu = 0.8 \end{pmatrix}$$

Among it $\lambda = k$, $k = 1, 2, \cdots 10$.

In order to a straightforward, put the result of $E(Q)$ as Paint 4.1.5, accurate data refer to **Appendix 7.2**. The changing situation we can see is:

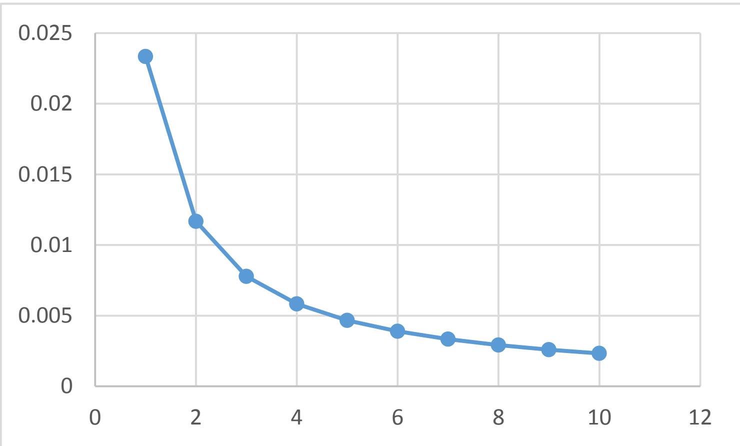

Figure 4.1.5 the Changing Situation of $E(Q)$ Along with the Arrival Rate Changes

This result is similar to the theory of the 4.1.1.

2. Simulation Results

Aiming at the inputting data, we simulate 1000 experiments in all. Taking $k = 3$ and $k = 5$ for example, put the results the program output as the **Figure 4.1.6** and **Figure 4.1.7**.

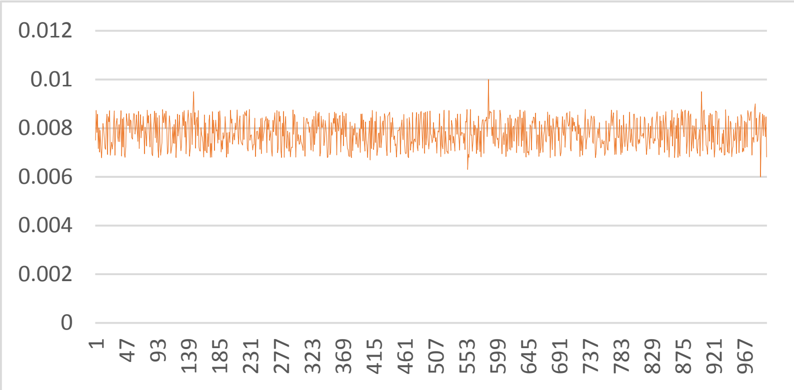

Figure 4.1.6 the Simulation of Results Fluctuation when $k = 3$

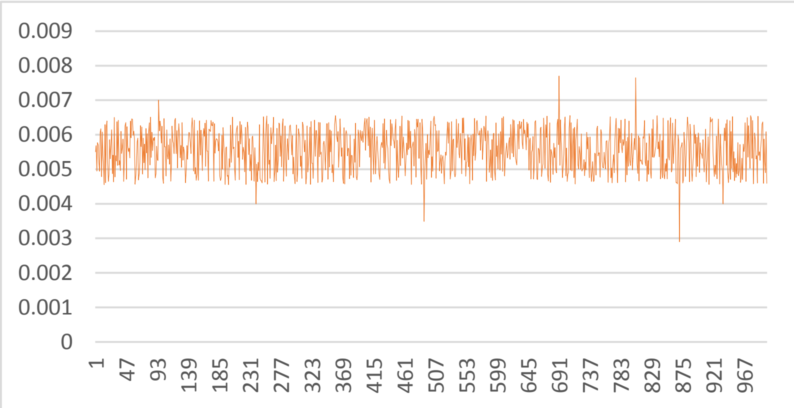

Figure 4.1.7 the Simulation of Results Fluctuation when $k = 5$

It is clear that simulation results are stable over a range, so we believe that the simulation results are coincide with the theoretical results.

The application also output the number of accidents in 1000 experiments. In order to reflect the simulation results directly, a graph is used to show the relation between the number of accidents and the vehicles passing the road section, as shown in the **Figure 4.1.8**.

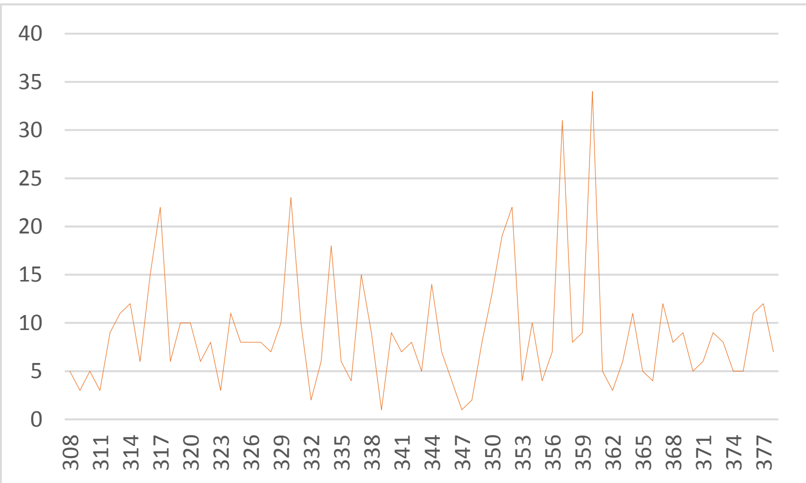

Figure 4.1.8 Relationship between Vehicles existed Security Problems and Traffic

As we can see in **Figure 4.1.8**, the number of hidden safety problems of the vehicle are rosing to about 20 and the fluctuation is not stable, so under the light traffic, there is dangerous drive vehicle on the highway.

4.1.3 to Discuss the Improvement Method about Keep-Left-Except-To-Pass

In order to improve traffic flow in the heavy traffic case, we need to put forward the improvement method of "Keep-Right-Except-To-Pass", our improved method is add the following rules in the "Keep-Right-Except-To-Pass" basis:

From left to right, setting carriageway in a certain speed gradient. At the same time, redefining the concept of the overtaking lane according to vehicle models. For example, large trucks can only use the right lane and only to overtake in the second right lane. Then for large trucks, the lane is overtaking lane, the left lane is this type of overtaking lane. The other case analogy.

Because this model is no longer applicable this set of traffic rules, we no longer make detailed theoretical analysis due to limited space, here only discuss the feasibility of a solution.

After adding some rules, vehicles will get a better classification on the road, for large trucks running slower, they are always in the right lane, there will not occur the phenomenon that randomly occupied the carriageway and subsequent vehicles can't overtake. In this case, not only to enhance the success rate of overtaking, but also enhance the traffic flow, but also convenient traffic management.

4.2 Keep-Left-Except-To-Pass Contrastive Analysis

The situation may change in the countries where it is required to drive on the left. Although the difference between driving on the left and on the right is only mirror symmetry, driving on the left theory should be identical to the right road. But from a series of researches in the literatures [5-7] we can come to the conclusion：

On the one hand, on the left side of the road, accident fatality rate can be reduced. In case of emergency, most people will instinctively tilted to the left or turn a direction.

On the other hand, when the lane change, there is a high probability of the driver which is moving to left lane rearview at left rear mirror and a high probability of the driver which is moving to right lane rearview at right rear mirror. That show that the driver mainly to obtain external information from the lane-changing, traffic information from the lane-changing is much more important. In the entire decision-making and implementation stage, the driver to obtain information more widely range and the driver's visual scanning is more flexible when they move a lane to right. Therefore, on the left side of the road, overtaking has better flexibility and security.

The causes above don't change the input data, but we can believe that the driver can precisely control the distance between two vehicles. As a result, the updated information of the road condition described in 3.1.2 should be adjusted subtly. We adjust the program to simulate the method for generating random values, the results as shown in **Figure 4.2.1**, details see **Appendix 7.2**.

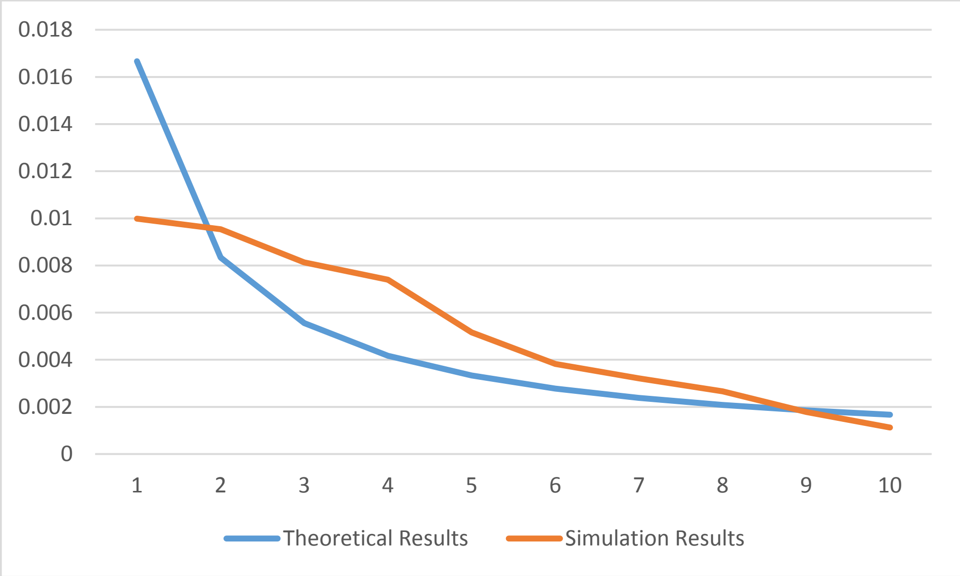

Figure 4.2.1 theoretical $E(Q)$ about on the left side of road contrast with Simulation

According to the results of 2000 experiments composed by the theoretical results and simulation results in 4.1, we believe that the theoretical results are reliable. Here we derive the theoretical results (as shown in **Figure 4.1.2**, details see **Appendix 7.2**), compared with the experimental results, it is not difficult to find:

The traffic flow under the condition of driving on the left is higher than that of driving on the right. Nonetheless, the number of vehicles driving on the left which has hidden danger is bigger.

4.3 Intelligent Control

In the intelligent control system, we can use the laser ranging system (LIDAR) to detect the distance between cars, In front of the vehicle speed and the speed of vehicles behind and a series of parameters, that is to do precise control when driving. Then, effects of some random variable model (the speed of vehicle overtaking, after the completion of the overtaking vehicle speed and so on) translated into the constants which is in the intelligent control.

At this moment, in our model, $E(Q)$ finally degenerate to the theoretical results. And based on our global assumption, there is no car which has safety problems.

At present, intelligent control system can always find the best driving solution, hence $E(Q)$ is a function only concerned with the speed limit $V^{(light,heavy)}$ section. And the bigger $V_{\min}^{(light,heavy)}+V_{\max}^{(light,heavy)}$ is, the bigger are the traffic flow and driving efficiency, that is:

$$\left(E(Q), Safety\_Index \to \infty\right) = TrafficProcess\left(V^{(light,heavy)}\right)$$

4.4 Conclusion

4.4.1 Question One

$$\left(E(Q), Safety\_Index\right) = TrafficProcess\left(L, P(C_i), V^{(light,heavy)}, t, \lambda, \mu\right)$$

(1) The calculation results of the theoretical models we built according to the driving rules "Keep-Right-Except-To-Pass" is matched with the results of the programming simulation, so the model we built has certain reliability.

(2) In the cases of light traffic, the number of cars which have safety problems is under ten, and the volatility of the cars is steady, so in the light-traffic cases, the safety is well.

(3) In the cases of heavy traffic, the numbers of cars which have safety problems increased to 20 or so. Compared to the light-traffic cases, the safety is failing, and is not very stable, so in the heavy-traffic cases, driving on the highway has a certain risk.

(4) Compare the theoretical $E(Q)$ of the light traffic with the theoretical $E(Q)$ of the heavy traffic, although intuitively the number of the cars is quite large, the Speed is limited because of the crowed traffic, and decrease the traffic flow.

(5) In order to increase the traffic flow in the heavy traffic, we come up with the improved method of "Keep-Right-Except-To-Pass". Based on the "Keep-Right-Except-To-Pass", increase the following rules: set the driveway according to a certain speed limit gradient from the left to right, at the same time, divide the various overtaking lanes according to the car types. For example, a large truck can only run on the rightist lane and use the second rightist lane to overtake, and so on.

4.4.2 Question Two

(1) This "Keep-Right-Except-To-Pass" driving standards cannot be simply changed to apply to those provisions driving on the left countries

(2) The traffic flow keeping to the left is higher than the traffic flow keeping to the right, but the number of the cars which have hidden trouble is also higher than the number of the cars keeping to the right.

4.4.3 Question Three

$$\left(E\left(Q\right), Safety\_Index \to \infty\right) = TrafficProcess\left(V^{(light,heavy)}\right)$$

(1) In our model, $E\left(Q\right)$ finally degenerate to the theoretical results. At this moment, the intelligent control system can always find the best driving strategy, and thus $E\left(Q\right)$ is just the function of the speed limit range $V^{(light,heavy)}$. And the bigger $V_{\min}^{(light,heavy)} + V_{\max}^{(light,heavy)}$ is, the bigger the traffic flow is, and the higher the driving efficiency is.

(2) Based on our global assumption, in the intelligent control system, there is no car which has hidden trouble.

V. Future Work

5.1 Strengths

- Simulation on the basis of theoretical analysis and programming point of view to analyze and contrast the results, increased the reliability of data.
- We have taken many parameters react the condition of the real traffic into account: test time t, road length L, car types i, numbers of occurrences $P\left(C_i\right)$, the speed limit range $V^{(light,heavy)}$, arrival rate λ, coefficient of friction μ.
- We almost ignored all potential safety hazard caused by human factor. It seems that it can't match the intelligent driving in the problem 3. But as a matter of fact, except the security hidden danger caused by human factor, the module shows that there are still vehicles with potential safety hazard. Consequently intelligent driving is much better than human driving.

5.2 Disadvantages

There are many control factors have joined the programming model, but the actual situation is more complicated. About Traffic accidents and road traffic conditions, there are many uncontrollable factors, such as the road condition, the driver malicious driving etc. If we want to simulate more accurate actual effect, we need to add more control variables which is not considered (such as the fuzzy factor that responses surface conditioned.) to simulate.

5.3 Trends and Perspectives

It has been successfully used for the unmanned vehicle to technology companies including Google as the representative, and the French company RobuRide, Holland Company ParkShuttle etc. Widespread use of this technology can not only prevent traffic accidents, and through the precise control of reducing carbon emissions from cars use. The more important point, also it can give people more leisure time.

However, there are still many problems cannot be ignored, the most controversial is the moral problem. In the present situation the development of science and technology, unmanned technology is still not mature. Unmanned technology is always regard the protection of personnel inside the vehicle and the vehicle as the first priority, but a driver may be willing to sacrifice his own car to protect others. For example, if the vehicle in front suddenly slip when you are driving, it's hard to stop. At this time, a big truck is on the left, most drivers would choose crashed into the side of the road and not mounted to the truck. But the unmanned technology will make what decision, we seem to also can’t predict [8].

Thus, the development of Unmanned Technology shoulders heavy responsibilities and has a long way to go. Only the true Human Intelligence is given control system, can it achieve the intelligent traffic system required safety, green, efficient requirements.

VI. References

[1] 谢军. 高速公路通行能力分析与服务质量评价研究[D]. Shaanxi: Chang'an University, 2007.
[2] 刘江等. 驾驶员气质与行车速度关系的初步研究[J] 北京工业大学学报, 2006,32(1):27-32.
[3] 荣建等. 基于可变跟驰时间和随机因素的通行能力理论计算模型[J]. 中国公路学报, 2001, 14(3):81-85.
[4] 魏朗等. 驾驶员道路安全感受模糊评判模型[J]. 交通运输工程学报, 2004, 4(1):102-105.
[5] Ke H. Study on driver's lane change behavior and law of eye movement in urban environment[D]. Xi'an:Chang'an University, 2010.
[6] Wikipedia. Right and left hand traffic[EB/OL]. [2014-2-7].
http://en.wikipedia.org/wiki/Right-_and_left-hand_traffic.
[7] 百度百科. 道路通行方向[EB/OL]. [2014-2-7].
http://baike.baidu.com/link?url=kg7xjpS-INfQO36YH99uY4D_vwUeSrlUgnVfTIChI6gKWOn7Us7ij4PAQ2k3MeyTp7q3pS2XieWIKyLgOyvSOa.
[8] 王嘉伟等. 浅谈智能交通与无人驾驶技术的发展展望[J]. TECHNOLOGICAL DEVELOPMENT OF ENTERPRISE, 2011, 30(14).
[9] law110.com. Design Code for Garage JGJ100-98[EB/OL]. [2014-2-7].
http://www.law110.com/lawserve/guihua/1800030.htm.

VII. Appendix

7.1 Code Kernel

```
//
//    Car.h
//    LaneSimulation
//
//    Created by Team# 28922 on 14-2-8.
//    Copyright (c) 2014 Team# 28922. All rights reserved.
//

#ifndef __LaneSimulation__Car__
#define __LaneSimulation__Car__

#include <iostream>

class Car
{
public:
        Car();
        ~Car();
        bool Overtaking();
        bool Changing();

        float length;
        float velocity;
        float first_headway;
        float last_headway;

};

#endif /* defined(__LaneSimulation__Car__) */

//
//    Lane.h
//    LaneSimulation
//
//    Created by Team# 28922 on 14-2-8.
//    Copyright (c) 2014 Team# 28922. All rights reserved.
//

#ifndef __LaneSimulation__Lane__
```

```
#define __LaneSimulation__Lane__

#include "Car.h"
#include <iostream>
#include <vector>

class TrafficLane
{
public:
	TrafficLane();
	~TrafficLane();
public:
	int getCarNum(){ return car_num; }
	void updateCarNum(int car_num){ this->car_num = car_num; }
	float getLaneUseRase(){ return lane_use_rase; }
	void updateLaneUseRase(float lane_use_rase){ this->lane_use_rase = lane_use_rase; }
private:
	int car_num;
	float lane_use_rase;
};

class OverTakingLane
{
public:
	OverTakingLane();
	~OverTakingLane();
public:
	int getCarNum(){ return car_num; }
	void updateCarNum(int car_num){ this->car_num = car_num; }
	float getLaneUseRase(){ return lane_use_rase; }
	void updateLaneUseRase(float lane_use_rase){ this->lane_use_rase = lane_use_rase; }
private:
	int car_num;
	float lane_use_rase;
};

class Lane
{
public:
	Lane();
	Lane(float length, std::vector<float> propability, std::vector<float> speed_range,
		float time, float arrive_rate, float miu);
	~Lane();
public:
```

```
    Car* cars;
    bool isNewCar();
    bool isSafe();
    float getTrafficFlow();
    bool updateRoad();
private:
    float total_time;
    float length;
    bool statu;
    float max_speed;
    float min_speed;
    float miu;
    int nusafe_num;
    float arrive_rate;
    std::vector<float> car_propability;
    TrafficLane *lane_traffic;
    OverTakingLane *lane_pass;
};

#endif /* defined(__LaneSimulation__Lane__) */

//
//  main.cpp
//  LaneSimulation
//
//  Created by Team# 28922 on 14-2-8.
//  Copyright (c) 2014 Team# 28922. All rights reserved.
//

#include "Car.h"
#include "Lane.h"
#include <iostream>


int main(int argc, const char * argv[])
{
    float lenth = 30;
    std::vector<float> propability = {0.15, 0.2, 0.3, 0.2, 0.1, 0.05};
    std::vector<float> speed_range = {90, 120};
    float total_time = 4;
    float arrive_rate = 1;
    float miu = 0.8;
    float t = 30;
```

```
    Lane lane(lenth, propability, speed_range, total_time, arrive_rate, miu);
    while (t+=0.01 != t) {
        lane.isNewCar();
        lane.cars->Changing();
        lane.cars->Overtaking();
        lane.updateRoad();
        if (!lane.isSafe())
            break;
    }
    return 0;
}
```

7.2 Data

Table 7.2.1 $E(Q)$ theoretical value ("right rule" light traffic)

Arrival Rate	Mean Traffic Flow
1	0.016667
2	0.008333
3	0.005556
4	0.004167
5	0.003333
6	0.002778
7	0.002381
8	0.002083
9	0.001852
10	0.001667

Table 7.2.2 $E(Q)$ theoretical value ("left rule" heavy traffic)

Arrival Rate	Mean Traffic Flow
1	0.023333
2	0.011667
3	0.007778
4	0.005833
5	0.004667
6	0.003889
7	0.003333
8	0.002917
9	0.002593
10	0.002333

Table 7.2.3 $E(Q)$ theoretical value ("right rule" light traffic)

Arrival Rate	Theoretical Results	Simulation Results
1	0.016667	0.009987
2	0.008333	0.009541
3	0.005556	0.008132
4	0.004167	0.007397
5	0.003333	0.005163
6	0.002778	0.003822
7	0.002381	0.003214
8	0.002083	0.002664
9	0.001852	0.001785
10	0.001667	0.001128